DEPARTMENT OF MATHEMATICS, UNIVERSITY OF PISA

A THREE-DAY DISPERSIVE MEETING IN PISA, 08-10 FEBRUARY 2024

: Event Webpage

BOOK OF ABSTRACTS

Paolo Antonelli

Gran Sasso Science Institute, L'Aquila

Dispersive estimates for the QHD system

Abstract. The quantum hydrodynamics (QHD) system is a prototypical model used in the study of quantum fluids. It describes the time evolution of a compressible, barotropic, inviscid fluid, subject to a stress tensor (describing quantum effects) depending on the mass density and its derivatives.

By means of the Madelung transformation, it is possible to establish a correspondence between solutions to nonlinear Schrödinger (NLS) equations and solutions to the QHD system. This in particular implies that such solutions naturally inherit some dispersive properties valid for the NLS evolution. However, not all QHD solutions can be determined by the NLS dynamics and in general it is not clear what is the class where this correspondence is one-to-one. This motivates the study of dispersive properties of the fluid dynamical system, without relying on the analogy given by the Madelung transformation. In this talk I will review some related results and try to discuss some questions in this direction that still appear to remain open.

Filippo Boni

Scuola Meridionale Superiore, Naples

NLS ground states on a hybrid plane

Abstract. In this talk we present recent results about the existence, the nonexistence, and the shape of the ground states of a Nonlinear Schrödinger Equation on a manifold called hybrid plane, that consists of a half-line whose origin is connected to a plane. The nonlinearity is of power type, focusing and subcritical. The energy is the sum of the Nonlinear Schrödinger energies with a contact interaction on the half-line and on the plane with an additional quadratic term that couples the two components. By ground state we mean every minimizer of the energy at a fixed mass. These are the first results for the Nonlinear Schrödinger Equation on a manifold of mixed dimensionality.
They have been obtained in collaboration with R. Adami, R. Carlone and L. Tentarelli

---

Serena Federico

University of Bologna

Unique continuation properties of variable coefficient Schrödinger equations

Abstract. In this talk we will discuss some unique continuation properties of certain space-variable coefficient Schrödinger equations in the Euclidean setting. We will see that, under suitable natural smallness assumptions on the coefficients, and assuming a certain exponential decay of the solution at two different times, the solution to the equation must be identically zero.
This talk is based on a joint work with Z. Li and X. Yu.

Filippo Giuliani

Polytechnic of Milan

Growth of Sobolev norms for the quantum hydrodynamic system

Abstract. In this talk we present a result of existence of solutions to the quantum hydrodynamic (QHD) system on the two dimensional torus which undergo an arbitrarily large growth of higher order Sobolev norms in polynomial times. These solutions stay close (in the sup-norm) to constant steady states, in particular they remain far from the vacuum over their whole lifespan. The proof is based on the connection between the QHD system and the cubic NLS equation, provided by the Madelung transform. We show that the cubic NLS equation on the two dimensional torus possesses solutions which starts close to plane waves and undergo an arbitrarily large growth of higher order Sobolev norms in polynomial times. This is an improvement of the result by Hani (Arch. Rat. Mech) and it is achieved by a refined normal form approach. Then we show that the existence of such solutions to NLS implies the existence of the solutions of the QHD system exhibiting a large growth in Sobolev norms.

Masayuki Hayashi

University of Pisa

Modified energies for the generalized derivative nonlinear Schrödinger equation

Abstract. We prove the global existence of H^2 solutions to the generalized derivative nonlinear Schrödinger equation on the torus. This answers an open problem posed by Ambrose and Simpson (2015). The key in the proof is to extract the terms that cause the problem in energy estimates and construct the modified energy so as to cancel them out by effectively using integration by parts and the equation.

Tomioka Kenta

University of Tokyo, Waseda

The vanishing dispersion limit of the Schrödinger-improved Boussinesq system and its first approximation

Abstract. We consider the vanishing dispersion limit of solutions for the Schrödinger-improved Boussinesq system (S-iB). Moreover, we consider the first approximation of the vanishing dispersion limit.
These are based on joint works with Professor Tohru Ozawa (Waseda University).

Felice Iandoli

Calabria University, Cosenza

Strong ill-posedness in L^∞ of the 2D Boussinesq equations

Abstract. In this talk I will present a recent work in which the strong ill-posedness of the two-dimensional Boussinesq system is proven. I will show explicit examples of initial data with vorticity and density gradient in $L^\infty(\mathbb{R}^2)$ for which the horizontal density gradient has a strong norm inflation in infinitesimal time.

Jessica Elisa Massetti

University of Rome, Tor Vergata

Long time behavior of Sobolev norms: Normal Form and Energy methods

Abstract. We discuss the problem of long time behavior of solutions of two given PDEs defined on a compact manifold. This talk will be twofold. On the one hand, I shall discuss exponential type stability times in the degenerate context of the beam equation with mass in 1 space dimension. A key ingredient is a suitable Diophantine condition (weaker than the original one proposed by Bourgain) that enables one to perform a Birkhoff Normal Form procedure. On the other hand, with the aim of relaxing the requirement on the size/regularity of initial data arising from the BNF, we discuss a different approach on the completely resonant NLS on tori. A key ingredient is some energy method based on paradifferential calculus and suitable tame estimates. The control over finite but long times on high Sobolev norms requires only conditions on the low ones.
This discussion is based on recent results in collaboration with Roberto Feola.

Boris Shakarov

University of Toulouse

Models for Dynamical Convergence to the Stationary States of the NLS equation

Abstract. We are interested in dynamic models for the approximation of the stationary states of the NLS equation. In particular, we will talk about a nonlinear parabolic equation that preserves the L^2-norm of the solution while dissipating the energy. This model is a continuous variant of the generalized gradient flow approach used in numerical simulations. After discussing the local and global well-posedness in the energy space, we will focus on the asymptotic behavior of solutions. We will show that solutions converge to stationary states of the NLS equation, up to a subsequence of times. We will then discuss convergence for all times. We will show that this convergence holds if the asymptotic state is a local minimizer of the energy.

SHULAMIT TERRACINA

University of Milan

REDUCIBILITY AND STABILITY FOR THE KLEIN GORDON EQUATION WITH A PERTURBATION OF MAXIMAL ORDER

Abstract. We prove the global in time existence and the stability of the solutions of a class of quasi-periodically forced linear wave equations on the circle of the form

$$u_{tt} - u_{xx} + mu + Q(\omega t)u = 0$$

where Q is an unbounded pseudo-differential operator of order 2, parity preserving and reversible, and the forcing frequency ω belongs to a Borel set of asymptotically full measure. This result is obtained by reducing the Klein-Gordon equation to constant coefficients, applying first a pseudo-differential normal form reduction and then a KAM diagonalization scheme. A central point is that the equation is equivalent to a first order pseudo-differential system which, at the highest order, is the sum of two backward/ forward transport equations, with non-constant coefficients, respectively on the subspaces of functions supported on positive/negative Fourier modes. The key idea is to straighten such operator through a novel quantitative Egorov analysis. A main point of interest, in view of applications to a nonlinear setting, is that the change of variables that reduce the equation satisfies tame bounds. This is a joint work with M. Berti, R. Feola and M. Procesi.

LEI WEI

Huazhong University of Science and Technology, Wuhan

DISPERSIVE DECAY ESTIMATES FOR THE MAGNETIC SCHRÖDINGER EQUATIONS

Abstract. We prove dispersive decay for the linear and nonlinear magnetic Schrödinger equations. For this, we construct the fractional distorted Fourier transforms with magnetic potentials and introduce the fractional differential operator $|J_A(t)|^s$. Combining the properties of the distorted Fourier transforms and the Strichartz estimates of $|J_A|^s u$, we obtain the dispersive bounds with the decay rate $t^{-\frac{n}{2}}$.
